Review of Contemporary Philosophy
ISSN: 1841-5261, e-ISSN: 2471-089X
Vol 22 (1), 2023
pp. 6780 - 6797

Fuzzy Logic analysis for managing Uncertain Situations

[1]Claudio Paya Santos, [2]Juan José Delgado Morán,[3]Luigi Martino,[4]Luis A. Garcia Segura,[5]Javier Diz-Casal,[6]Juan Carlos Fernández-Rodríguez

[1]Valencia International University, Spain

claudio.paya@professor.universidadviu.com

https://orcid.org/0000-0002-1908-9960

[2]University Pablo de Olavide, Spain

jjdelmor@upo.es

https://orcid.org/0000-0002-9945-8235

[3]Bologna University, Italy

luigi.martino3@unibo.it.

https://orcid.org/0000-0002-7417-2898

[4]Nebrija University, Spain

lgarcise@nebrija.es

https://orcid.org/0000-0001-7074-4396

[5]Alfonso X el Sabio University, Spain

jdizcas@uax.es

https://orcid.org/0000-0003-1332-8905

[6]Universidad Atlántico Medio, Spain

juancarlos.fernandez@pdi.atlanticomedio.es

https://orcid.org/0000-0003-3312-861X

Abstract

The analysis of fuzzy logic is fundamental for managing states of uncertainty, allowing the representation of variables that are neither completely true nor completely false. This logic is based on the manipulation of fuzzy sets, which enables the modeling of situations where information is imprecise or vaguely defined. In the context of risk management, fuzzy logic is used to make more informed decisions under ambiguity, facilitating the evaluation and selection of alternatives through linguistic terms and assessment scales that reflect human perception. These methods are applicable in various fields, such as information auditing, education, and business decision-making, where uncertainty plays a crucial role

Keyword: Fuzzy Logic, Uncertainty Management, Decision Making, Decision Strategies, Ambiguity

1. Introduction

In daily life, human beings often find themselves in states of uncertainty, for example, when trying to assess a situation or estimate future trends. Similar uncertainty states also arise in many applications of artificial intelligence (AI) and expert systems, for example, when analyzing vague and uncertain propositions. In the literature, uncertainty is classified into two main categories, which are based on differing world views. To be complete, these conflicting views are given very briefly. The first represents the formalist view based on mathematics. Knowledge is conceived as a set of structured propositions that are either true or false. Truth is represented by probability measures or numerical parameters between 0 and 1. Inference consists of means for obtaining new knowledge from these propositions based on certain probabilities .

Often a proposition is not interpreted truthfully, but vague and uncertain. Uncertainty can arise from the vagueness of propositions, from merely limited knowledge, and from incomplete knowledge. The second category represents the view where knowledge is conceived as a conceptual entity. Knowledge cannot be fully expressed by a definite set of propositions. An imprecise proposition subsumes a set of similar opinions, which are intuitively captured by vague concepts. Therefore, vague knowledge bases are more concise and descriptive. Linguistic variables and fuzzy logic were proposed for the representation and processing of such vague and imprecise knowledge. However, propositions can be consistently interpreted by means of formalism .

Incorporating fuzzy knowledge into expert systems poses difficult problems with high complexity. To enhance their acceptance, more attention has to be given to the handling of vagueness and uncertainty. Currently, experts are supported with decision support systems (DSS). The crucial knowledge representation framework for most DSS is a knowledge base containing rules. However, hardly any knowledge is represented formally and merely implicitly. So far, no rigorous approaches have been proposed to assist the semantic determination of vague expert rules. Alternatively, a framework for uncertainty management is proposed that was developed for fuzzy decision support systems. Such systems do not require a sound representation of fuzzy knowledge. The domain expert's knowledge is expressed in terms of a set of rules, which frequently refer to vague and uncertain propositions.

2. Overview of Fuzzy Logic

Fuzzy Logic Attempts to Solve an Uncertain Problem

The traditional logic suggests an element is useful and acceptable for the operation of existing systems with the rules of 0's and 1's based on Boolean logic. The information is assumed to be a truth and very precise with full accuracy. On the other hand, fuzzy logic tries to solve such relationships based on various degrees of acceptability uncertain about new or modified systems. Fuzzy information is said to be of the vague or imprecise like hot, height, near, etc., based on membership, knowledge, and reasoning. The data may have characteristics like skewness, broadness, uncertainty, ambiguity, incompleteness, etc., may affect the decision-based correctness of the information. The degree fuzzy membership between 0 and 1 may model the uncertain systems and relationships better than the theories of precision difference. The fuzzy systems provide the degrees with which a fuzzy logic element belongs to some predefined sets. Using fuzzy IF-THEN rules, the fuzzy model provides the fuzzy degrees for each IF-THEN rule. The answer may not be output crisp, and hence defuzzification processes are performed to find the solution complete information. A new membership function in such input, output, and inferred degrees may aggregate uncertain data fuzzy or otherwise.

Possibility theory and Bayes theory are useful to the formalism of fuzzy uncertain data, but the reasoning results of systems with uncertain degrees may have additional constraints. In view of such complexity in the fuzzy systems, the behaviour of fuzzy logic controllers with conventional prescribed knowledge bases may be either impractical or unfavourable. In most cases, the performance robustness and reliability with crisp noisy measurements of systems with new designs are critical in the control flow. The fuzzy uncertain data are both fuzzy and normal. The uncertain information contains one layer of fuzziness and another layer of randomness. The uncertain systems were fuzzy in the sense that a fuzzy value is assigned to every normal variable. Various solution approaches like fuzzy Kalman filters, centroidal fuzzy observers, differential equations, adaptive filters, fuzzy sliding-mode observers, etc., are also available for the tractability of control applications. Presently, eagerly modified solutions to the tracking of exhaustive derivatives for designing and implementation on chips have approached fuzzy observers. In controller design, model-free neuro-fuzzy attempts were undertaken on the behaviour of mobile platforms.

In an attempt to linearize the underlying uncertain systems, various soft/multiresolution synthesis modelling methods are available, with rule extractions being discussed with limited examples. The simplest number connected with fuzzy number arithmetic operations and reassembling of nodes for quantitative performance evaluation was presented. However, logic operators were not directly associated with such arithmetic degree-2 information aggregation measures for approximate reasoning. Effectiveness did not depend on the membership function with membership and alpha cuts, allowing a focus on the concept of type-2 fuzzy numbers and fuzzy clustering of numbers. Complex arithmetic operations may be solvable in implementation to find controlled or intelligent solutions with fuzzy information.

2.1. History and Development

In a recent introductory paper (Virgil Negoita, 2011) there were presented the motives who made this analysis of fuzzy logic possible and there was explained the approach (first as a complementary analysis of facts and just afterwards a systematization attempt for all aspects). The aim of this analysis is to present the fuzzy paradigm w.r.t. the knowledge about it, as well as the tools own to any paradigm and, especially, to any paradigm of a relatively new scientific area (as is the area of the modes of management of uncertainty states). The flexibility and the openness characterizing fuzzy set theory and fuzzy logic is in congruence with the claims twelve years ago, knowing that later, as it was hypothesized, the development of fuzzy crisis was possible (Parvanov Angelov, 2019).

With the aims, the working hypotheses, the limits of space and time of this analysis established, the efforts were concentrated upon looking for languages and models, which can allow for a non-numeric representation of concepts and modes of functioning of fuzzy sets and systems, and which would be claimed to be “natural”, “ordinary”, commonly used linguistic constructs (being mathematical models devoid of rigor). The development of fuzzy sets and systems popularized through books and papers is huge, being amplified in the last decades (by the introduction of fuzzy knowledge base systems, fuzzy automata, fuzzy composites, logic programming control, etc.). Some of these huge development need a deeper analysis, investigation, and thought for their understanding and for their completion. In many cases, this is only for gaining knowledge about fuzzy systems as an all-embracing class (one-to-one functional relation between inputs and outputs), because the other tasks and works (especially concerning the modelling of fuzziness) are related to a narrower class.

2.2. Fundamental Concepts

Fuzzy logic, as initially described by Zadeh, was meant to model human reasoning, specifically its capability of reasoning with uncertainty. Its ability to handle uncertainty about premises has proven a powerful aspect of fuzzy logic that makes it possible to model many human cognitive and reasoning mechanisms. However, this is not its only important aspect. There is still the possibility of having uncertainty about a degree of membership, and this has been addressed with different approaches from the one usually adopted. This remark has a fundamental impact on the subsequent development of fuzzy systems and their applications. Indeed, it is one thing to have statements such as “the temperature is medium” such that there is no uncertainty about the degree of membership of the temperature to the medium set, and it is another to have statements with uncertainty. This is similar to the difference between crisp or standard fuzzy logic and possibilistic logic on the truth value side of those languages (H. Ruspini, 2013).

One fundamental difference between fuzzy systems and classical discrete systems is about how information is represented. In the case of a classical discrete system, the precise classification of an object allows dealing with it as one and only one prototype. In the case of fuzzy systems, the situation is very different due to the vagueness of the concept employed in the description of the object. Vagueness necessarily leads to extreme situations where there is uncertainty about the classification of the object being treated (D’Alterio et al., 2018). Even in the cases where precision is required, as in control systems, vagueness and approximation can enhance the fuzzification of states to smooth step-control operations.

2.3. Applications in Various Fields

Fuzzy Applications in Business and Economics. Many of the problems occurring in business and economics are rather complex and hence difficult to be solved precisely. Therefore, conventional methods often fail. Fuzzy methods introduced approximately 30 years ago may help admirably. One very important field lies in the storage and handling of knowledge in databases. In this connection, numerical databases and the possibility of classical logical conclusions cannot be developed there (Meyer & Zimmermann, 2011).

Fuzzy Application in Decision Theory. In decision problems usually one evaluates many alternative possible answers and looks for the one most suitable in view of maximal precision. However, many decision problems arise in situations in which one is looking for a decision criterion which ensures a semblance of harmony between all relevant arguments. Content-related applications can be found in non-technical areas such as economics, environmental protection, business administration, social sciences, etc. Decision support systems in environmental protection tend to focus on the design of public laws. Fuzzy models can be of great assistance in this connection.

Fuzzy Applications in Control. Since around 1970, the new and globally understandable 'fuzzy' was defending its potential against efficiency and precision. Technologies which based on fuzzy soft literature started to penetrate different markets and to claim part of the big gain and profit. The fuzzy applications in industry started with large standardized plants like reactor technology. This was the time of the fuzzy smoke detectors or the fuzzy controls of the washing machines and refrigerators. Afterwards, fuzzy industries recognize in a more and more agile world the need of flexibility in their operations (Chaudhuri et al., 2013).

3. Uncertainty in Decision-Making

Decision involving uncertainty is a central theme in the theory of decisions. Businesses and markets are made up of decision makers who procure, model, analyze, plan, and decide in situations reflecting uncertainty. It is well-established through empirical evidence that decisions made by a person of experience are more reliable than those made by taking into account only explicit variables (sometimes termed objective). It is also common to observe that individuals face situations of uncertainty in which the probability of occurrence of future situations is difficult to estimate. In such cases it is desirable that the environmental knowledge and experience of decision makers should be made a part of the decision process. In most of the real life problems faced during business decision-making, the input information available to decision makers is imprecise or uncertain (Keropyan & Maria Gil Lafuente, 2011). Fuzzy logic has established itself as a compelling pragmatic method for modeling the knowledge and actions of systems, particularly in applications that have critical control implications. It has provided tracking control systems for Earth correspondence with artificial satellites whose dynamics are geostationary, with an accuracy in the order of seconds at distances between them and the Earth of 36,000 km.

Fuzzy logic can also be used for managerial decision-making. It can be viewed as a tool for logically structured reasoning, in which one can express input information based on subjective rather than strict scales. On the other hand, descriptions, and consequent reasoning can be close to how humans pass judgment, allowing for qualitative reasoning comparable to humans. This is particularly useful and to some extent, necessary in strategic management where qualitative aspects explained through fuzzy logic play central roles in the modeling. A decision model framework based on fuzzy methodology is shown (Oluseyi Oderanti, 2013). This framework encompasses linguistic variables based on fuzzy numbers and weight determination based on priority vectors derived from pairwise comparisons between the alternatives. The decision maker's acceptance of the pairs and the corresponding weights of the pairs do not have to be precisely defined. Rather, a set of weights in fuzzy terms is adequate, which captures the decision maker's knowledge and experience in compliance with the terms of reference, the project scope, or similar documents.

3.1. Types of Uncertainty

In the field of fuzzy logic systems, it is assumed that the world perception by agents is never limited to binary situations but always presents some degree of membership in the true and false sets. Accordingly, the natural approach to this task is to model this perception with fuzzy logic and the corresponding fuzzy knowledge bases. The first significant limitation of the traditional fuzzy logic to manage knowledge is that the inputs are expected to be crisp. In the true world perception process, there are some limited imprecisions in the measurement systems and information gathering techniques. In such cases, the traditional fuzzy logic systems give rise to some performance fails. Taking into consideration the level of imprecision in the input fuzzy sets has been proposed. The alternative approach is to assume that the fuzzy sets are of type-2 (Reza Hassanzadeh, 2016). With the assumption that the uncertain levels, i.e., the second level of fuzziness, are themselves fuzzy, the fuzzy set is denoted again as type-3 (S. Khuman, 2016). Essentially, uncertainty might be classified as either epistemic or aleatory states. Epistemic uncertainty may exist when there is a gap in the available knowledge with respect to a fact, event or observable. It is of a subjective nature since it depends on the knowledge available to each individual or group and could be reduced by refining the knowledge or obtaining further evidence. On the other hand, aleatory uncertainty exists when a phenomenon is inherently unpredictable. In this case uncertainty is "objective" and cannot be reduced by improving data or models, but only by using models at a different level of resolution. For numerical modelling, the uncertain variables are referred to as uncertain parameters and the variables related to proper deterministic descriptors are termed as uncertain functions. The fact that designed models are other than perfect – even when the parameter values are known with infinite precision – leads to uncertainty or divergence in the computed values of quantities of interest. Such uncertainty is referred to as model uncertainty.

3.2. Impact of Uncertainty on Decision Processes

In a decision-making process, there is always uncertainty on several states. Broadly, we can classify uncertainty into two major groups: structural uncertainty and uncertain states. Structural uncertainty is associated with the system in which the processes take place—what actions can be performed? how are they performed?—and is often modeled through the use of different types of graphs and trees. On the other hand, uncertain states are those situations in which the system has not been precisely determined from the information available (Keropyan & Maria Gil Lafuente, 2011). All the uncertainties about the situation fall into this group. This paper deals with the latter type of uncertainty in decision-making processes. The basic characteristics of uncertain states are firstly discussed, together with a review of existing uncertainty formal representations. Then, some characteristics of a fuzzy approach to their representation are considered. Finally, the system architecture to manage uncertainty is described with some required adaptations and extensions, and the design framework for a specific management approach.

Fuzzy logic is a problem solving and knowledge representation framework that allows to treat imprecision and vagueness, it grants mathematical rigor to the analysis, informational support for the generation of the model, and well-documented existing implementations. It has been successfully applied in many domains, and its importance is increasingly greater as the complexity of the systems increases. It is extremely important in the social sciences, where uncertainty is high and human and social precepts vague and inaccurate. Decision processes in business and management typically qualify these characteristics, thus regarding fuzzy modeling not only as a possibility but also as a need (Oluseyi Oderanti, 2013). Such decision processes are subject to uncertainty on the states of a changing environment whose outcomes are implicitly modeled by the structure of a graph. Existing models of the problem do already represent states by fuzzy sets, the concept of fuzzy state (as intended in the present work) is not explicitly taken into account and because of this essential consideration the models suffer from an intrinsic lack of generality.

4. Fuzzy Logic vs. Traditional Logic

Traditional logic is an extension of Aristotelian reasoning, being a symbolic language of discrete or exact truth-values 0 and 1. A proposition may be either true or false, with nothing in between. It captures a deterministic point of view of the world, describing things by well-defined concepts and certainties. Traditional logic interprets sentences univocally, assigning them one unique truth-value. Regarding uncertain states, as probability cannot assess vagueness or ambiguity, the classic logical system cannot be extended to a fuzzy one.

Fuzzy logic is an approximate reasoning with degrees of validity. Fuzzy logic and fuzzy set theory were proposed and soon spread everywhere. Fuzzy systems consist of a fuzzy inference system, an adaptive/fuzzy interface, and a defuzzifier. Fuzzy systems accentuate the rough aspects of numerical models, by means of approximate reasoning and fuzzy representations. In complex problems, focus on little data and knowledge is used as a basis for more effective models. Besides applications, fuzzy systems are becoming design tools. Fuzzy set predicates can represent themselves in fuzzy rules. Subsystems with fuzzy output can be combined with neural networks.

Both fuzzy logic systems and neural networks are employed extensively in engineering, medicine, finance, and industrial process control. A variety of experts with distinctive backgrounds, areas of knowledge, experience, and heuristic understanding can be integrated into an advanced fuzzy-neural environment. Last decades witnessed fuzzy systems being used in safety and risk management. Knowledge and experiences regarded as subjective have been embedded in input-output models via fuzzy if-then rules. There has been little research focusing on neuro-fuzzy systems to assess the safety and risk management of landfill ordinal data. A sound and rigorous modelling methodology for both crisp and fuzzy evaluation of the landfill risk and safety assessment has been developed, as well as an explanation of its mathematical foundations.

4.1. Comparison of Approaches

A comparison of the two basic approaches to uncertainty management in fuzzy DEX is carried out, the capability of which has been illustrated using two artificial vowel-based applications and a real-world application in the field of investment portfolio selection. As an instance of a rule-based fuzzy model, a fuzzy network is enhanced with the capability of fuzzy uncertainty management. Having this capability, a fuzzy network can be used for network type reasoning over fuzzy or linguistic DEXs considering or not considering imprecise uncertainty regarding input fuzzy propositions. Several methods for aggregation of fuzzy univariate memberships that are valid under either probabilistic or degree-of-belief semantics are generalized for fuzzy DEX. These methods make it able to use fuzzy DEX for network type reasoning with input fuzzy propagations either generated by fuzzy information discretization methods or coming from fuzzy DEX as the output of a fuzzy-logic processor. Fuzzy DEX is proven to be able to use either social choice or deterministic procedures for the aggregation of judgments. Means for comparison of the performance of fuzzy logic composed of aggregation procedures using the decision aid notion of parameter-rich index of closeness as well as adjusted robustness criterion are proposed. The capabilities of fuzzy DEX, fuzzy aggregate DEX and fuzzy networks have been illustrated with some examples of artificial and real-world fuzzy applications.

Fuzzy logic is introduced as a concept, including its background mathematical theory. The components of an FL system and the makeup of generic fuzzy inference systems are discussed. As an alternative to the conventional perspective emphasized hereupon, recent use of fuzzy reasoning as a less pickier alternative to black-boxing linear approximation decision trees using rule pruning/adding and ad hoc rules is also illustrated. An overall commentary on the state of the art of fuzzy modelling and future challenges including interconnections between fuzzy and qualitative modelling is proposed. Original and comprehensive components of FL as rational tools for reasoning under conditions of uncertainty may be summarized as follows. One and older of the two components of fuzzy logic is the attractive yet practical concept of computing with not necessarily accurate degrees-of-truth. The other and newer component of residential estimation functions of FL is relative independence from the geometry of the underlying universe. A few fuzzy and uncertain academic research horizons regarding open questions of fuzzy logic as a degree-of-truth based, rational coping with vagueness and fuzziness may be outlined: One concerns rigorous treatment of fuzzy operators on non-discrete universes. The other open questions concern issues regarding FL and intelligent agents including hybrid closed formulae.

4.2. Advantages of Fuzzy Logic

Fuzzy logic provides distinct advantages as compared to the traditional methods. It has been successfully used for uncertainty treatment in various applications such as digital communication, medical diagnosis, control, manufacturing, pattern recognition, etc. Some points may clarify these advantages of fuzzy logic for uncertain management. Fuzzy rules are able to represent knowledge specifically, exactly and accurately since it can well express the vague and qualitative nature of human experts (Ridvan Benatar, 1970). Methodologically, fuzzy set theory can transform knowledge from qualitative to quantitative and thus different kinds of uncertain knowledge can be treated uniformly. Fuzzy logic is very flexible; for example, a fuzzy model can be easily modified if new or better information is obtained. Compared with the precise model, the fuzzy model can be constructed very cooperatively. If the fuzzy rules are constructed well, they can be easily verified by human experts. It is straightforward to figure out the fuzzy boundaries for which the outputs will not be affected.

The degree of uncertainty of a fact can be measured better in the fuzzy way than in the probabilistic way. Chance or classical uncertainty is associated with unforeseen but fully known occurrences. That is, if one can use the chance set well, no unexpected facts may happen. The bias or fuzzy uncertainty is associated with unforeseen and partially known occurrences. That is, no matter how well a chance set is used, surprise occurrences may still appear. Fuzzy logic can not only well treat biases but also manage fictitiousness. Then if there are no bias rays, with an appropriate normalization scale, the fuzzy sets should exist.

The basic difference between fuzzy logic and standard logic can be stated as ‘at what level is a fact true?’ comparing with the ‘Which facts are true?’. Consequently, fuzzy logic can be applied to handle fuzziness but no degree is intended when using standard logic. In many reasons and rules fuzzy logic encodes such degrees while usually not in the relations and operations (Kerarmi et al., 2022).

5. Fuzzy Sets and Membership Functions

The object of fuzzy set theory is characterized by a membership function (mf) with a range of the real unit interval [0, 1]. Linguistic fuzzy sets (LFS) such as “young,” “medium,” and “old” can be expressed as fuzzy sets. The classic categories of a subject of interest in standard (crisp) set theory, on the other hand, can be described in black and white. As a result, fuzzy sets can reflect the qualitative gap of a field more precisely than classical sets. On the other hand, fuzzy membership functions (mf) are not only able to assess unfuzzy fuzzy sets, but they have been employed in numerous applications for capturing the states of noisy data, handling vagueness, and correcting misrepresentation in data. As a predominant classification tool, this local-to-global connectivity is frequently used recently. The gravitation principle can be restated in a different formulation, However, it is stated that membership functions operate on intervals of possible intervals of possible states description and that, when uncertainty is accepted, the evolution of an interval input is again an interval output for interval operations, a metamorphic extension of a LFS is also readily designed (Jamali & J. Bani, 2017). Subsequently, membership functions can be deployed on LFS. Ideally, inferences are performed on the LFSs resulting from mf designs. However, all input-output states and target membership functions are in actuality fuzzy. In order for the system to be a fuzzy logic controller (FLC), the input state quantization must in-line with the membership function combination described on LFS sets. The conciseness of fuzzy inference rules can be unsatisfactory due to the exponentially increasing number of rules with respect to the lines of input variables (Terdpravat, 2004). Thus, to effectively reduce the number of fuzzy rules, some methods have been developed.

5.1. Definition and Properties

Referred concepts of fuzzy descriptions, fuzzy implications, fuzzy partitions, and fuzzy set have to be formally defined before introducing properties of fuzzy logic. If H denotes a universe of discourse, then fuzzy membership functions on H are denoted by A : H→[0,1]. Fuzzy descriptions BG of objects in a conceptual region G⊆H are obtained when a partition C of the universe is chosen. Each fuzzy class B consists of the set of fuzzy partition associated with its respective fuzzy descriptor. The fuzzy implication among the fuzzy class is defined as a doubly-sided fuzzy relation from the fuzzy decision region to the fuzzy outcome region. Fuzzy reasoning is defined on the fuzzy classification by obtaining a fuzzy class or a fuzzy membership function for an object which does not belong to either partition. Handling a non-fuzzy input in fuzzy reasoning requires introducing a resolution operator called fuzzy partition. Then the reasoning on a fuzzy class can be characterized by recursively applying the fuzzy implication on local partitions of partition fuzzy regions. This approach employs the basic types of fuzzy logic including type 1, type 2, some intuitionistic fuzzy logics, and some fuzzy modal logics (F. Eick, 2013). A new theoretical framework integrating fuzzy logic and random phenomenon modeling on a general universal description base is further suggested. This approach will provide new computational methods for uncertainty reasoning task capturing event structures of mixed types in a weighted first order belief language whose weights are either fuzzy sets or conditioned probability distributions.

5.2. Types of Membership Functions

Mathematically, given a delineating set A on R, the fuzzy set A with the membership function One way of obtaining truth values of fuzzy propositions is through simulation of fuzzy selector S used in systems. The fuzzy selector yields membership degrees, or bucket clips Lu on set U, defined as possible values of a controlled quantitative variable X. Such numbers may be constructed from the output of a simulated fuzzy intervals based on rules. Alerts should be associated with the possible values (tracing or countersinking the input change state on the level of events). They are provided by calculating sums of fuzzy intervals (LU) created with respect to the alarms and their originating quantified variables.

Fuzzy set F(X) with membership function (m.f.) A(S) where A:- fuzzy selector S. Propositively, Fuzzy selector S with membership function Nu=mdt(S) evaluated for vertex points. Quantitative inferences: Deviations of system states (position of variable X) from reference values (set of snapshots) trigger Fuzzy selectors processing the input samples within time window. All Fuzzy selectors yield membership degrees (A {xi}) with respect to the asked bucket clips U (possible values of controlled variable X) through: A cumulative disjunction, or union of alert fuzzy sets, Us. Methodological point of view on the fuzzy approach to comprehensive uncertainty management in complex systems is focused on two thematically distinct fuzzy constructs.

First, well known experience to fuzzy selection of input information for the logical inference is discussed. The aim is to form fuzzy propositions and to construct the truth value given the fuzzy and quantitative variables, respectively. The precedence is given to designing a fuzzy selector. The selection is to determine the determination of the truth value and the temporal spreading of fuzzy events. This way the fuzzy outputs become determined fuzzy set. The propositions can be later evaluated and used through a procedural game-states-decisions action than fuzzy inference. Quantitative inferences by selecting bucket clips U, Fuzzy control is to trigger triggers based on the temporal excursions of numerical (optional, quantitative) variable X to some values b, which, after definition, form state (fuzzy) alerts. The adherence degree to squat settings (b) is evaluated by fuzzy membership degree $0<mU<1$. The alerts (when their membership degrees are heightened) are summed, evaluated, processed, and aggregated through disjunction in a “one rule” fuzzy control.

6. Fuzzy Inference Systems

Fuzzy Inference Systems (FISs) are computing frameworks based on the concepts of fuzzy set theory, fuzzy IF-THEN rules, and fuzzy reasoning. An FIS is a system comprising the rules, the data, and the procedures for manipulating them. A FIS model consists of inputs represented by fuzzy membership functions and a collection of fuzzy IF-THEN rules. The output of the FIS model falls within a specified range. The number of fuzzy rules can be determined based on the input space. Each fuzzy rule is represented by an index. The antecedent or the consequent of the fuzzy rules can be modified by a linear mapping, while the output can be treated as an aggregation or a defuzzification operator (Meng Tay, 2011).

Fuzzy reasoning refers to the procedures and techniques for a FIS to perform a given task specified in terms of fuzzy rules. For a standardized fuzzy reasoning, the premise or the act part of the fuzzy rules are relied upon which are computed separately before aggregating. Each fuzzy rule is considered to be independent of others. Many FISs adopt the view of standard fuzzy reasoning as the prototypical way of performing fuzzy reasoning and ignore alternative ways of reasoning with fuzzy rules (F. Eick, 2013). Fuzzy reasoning is a complex process where a FIS takes as input a number of measurements from the system to be modeled. The crux of an FIS lies in how the rules, which usually are vague and uncertain, infer a crisp prediction. Each fuzzy rule of the FIS captures an expert's knowledge about the input/output relationship. The input conditions are modeled in terms of fuzzy sets. Each fuzzy rule is evaluated using fuzzy implication to obtain the fired strength, which reflects the extent of certainty to which the rule is satisfied. The prediction mechanism takes as input the fired strengths of rules to aggregate the output predictions, which in return fuzzifies knowledge in terms of fuzzy sets.

6.1. Structure and Components

The component structure includes model variables, estimation modules, a rule base, inference mechanisms, and a decision base (similar to static fuzzy models) and adds the implementation of knowledge acquisition. Dialog control and the data containment functions also remain unchanged. Other components such as graphical display, data selection, and communication support may depend on applications. However, unlike static fuzzy models, it uses an agenda with ranking and queuing capabilities to explore added dialogue functions, such as layer computing, discrimination testing, and reverse reasoning. Also, in some places, special mechanisms (performance evaluation of rules, automatic accumulation of measures into rules, etc.) recommend those performance evaluation mechanisms (F. Eick, 2013).

The knowledge representation structure complements the static knowledge representation (in which all fuzzy model variables and SQL estimators are defined). In addition to the static part, the dynamic representation allows the storage of knowledge that may be added, modified, or instead deleted during the dialog. Dynamic representation could use various techniques such as explicit lists, meta-rules, or more complicated strategies. Regardless, certain representation functions, retrieval mechanisms, and maintenance techniques must be developed and made available to use any technique. The efforts are centered on explicit lists due to their straightforwardness. In terms of additional data structures, double entry parameter matrices create simple equivalent data representation for an event's witness. Dynamic representation includes: Static parameters are generally consistent with and observed in the same way as fuzzy models, and also changed concurrently within the same cycles or sessions (H. Ruspini, 2013).

Interactive variables or related valuation estimates can be added along with fuzzy model variables. Additionally, prior transformation problems are generally similar for interactive design. Static parameters and states (memory or continuity variables) are generally pre-defined, and their inclusion is not a major development. However, such dynamic events may be involved in the dialog, which is not built in proactive static fuzzy models. Thus, they should be captured separately and applied to modification capabilities by dynamic management techniques.

6.2. Types of Inference Mechanisms

The inference mechanism constitutes the rules from which to draw new conclusions based on a set of initial situations. A consequence of the nature of fuzzy systems is that inference is not guaranteed to be deterministic: the initial fuzzy sets and the rules can yield an enlarged fuzzy set in the target space, without a scope of crisp value deduced from the knowledge base. This is one of the key differences with traditional expert systems: obtained results are to a degree uncertain as well (F. Eick, 2013).

To manage both types of uncertainty, fuzzy systems embed their own uncertainty inside a specific logic that ensures wide fuzzification of the results. Whenever new evidence is introduced, an inference function is called in the same logic, processing the new evidence into support distributions. Then, a simple combination is performed at the output fuzzy set at a higher level of uncertainty, meaning that the uncertainty of the results is likely enlarged (D'Alterio et al., 2018).

Every fuzzy inference system is specified by a control structure of rules in terms of a particular form of the membership function, disambiguation, fuzzification, quantification, aggregation, normalization, and defuzzification function. Several types of fuzzy inference systems exist; they differ essentially in the choice of each of the control parameters.

A fuzzy inference is a mapping from a given fuzzy antecedent set to a fuzzy consequent set based on fuzzy IF-THEN rules. A mapping rule permits the generation of a fuzzy consequent set by taking into consideration a given fuzzy antecedent set. If fuzzy IF-THEN rules with the same forms and different parameters exist, the fuzzy mapping rule becomes a fuzzy inference rule, which is represented in a general framework of fuzzy if-then rules. The mapping fuzzy relation exists between a fuzzy input set and a fuzzy output set in a fuzzy inference. When considering the mapping fuzzy relation in interval type-2 fuzzy inference, the degree of a type-1 fuzzy set being incorporated in the mapping relation is uncertain.

7. Fuzzy Logic Applications in Management

Fuzzy technology applications in marketing. Marketing is strongly oriented towards customer needs. Customers have various needs. At first customers often do not know what kind of product they want or need. Qualifying and quantifying customer needs can be difficult. Potential customer needs can range from linguistic fuzzy statements, such as high price, latest technology, and fast performance, to numerical needs, such as price less than $800 and response time less than 1 ms. Linguistic human needs have to be quantified, numerical customer needs have to be expanded for the product's target specification. Assessment models transform customer needs into a set of target specification profiles. These profiles consist of value points for degrees of membership of a collection of linguistic sets. It might also be of interest to find a configuration of specification elements that matches a customer's need. An optimization model serves this purpose. Finally products matching the customer needs have to be searched for a fuzzy product database. Fuzzy search models filter those product profiles that match the target specification profiles as good as possible (Meyer & Zimmermann, 2011). Fuzzy Logic Applications in Production. Press formation in papermaking - production control of a succession of two stage processes - is an on-line application of fuzzy controllers. Fuzzy controllers control the press nip loads of a press section to reduce pre-felt erosion and sheet properties variability within a production run of 15,000 tons of paper. Drying control in a paper machine is an example of a very large and very complex off-line fuzzy logic application. An expert system coded in a fuzzy formalism controls quality control in press and dryer sections of paper machines. The expert system's fuzzy controls are displayed and evaluated in a live visual display.

7.1. Risk Management

In projects, variability will appear in frictional states during the execution of work; so, what can be done in project planning during its execution to minimize their impacts, is addressed in (Jones, 2001). Two fundamental problems that determine and illustrate what steps can be taken to minimize impacts of frictional states, such as re-planning, re-evaluating resources, are proposed as a fuzzy logic/mathematical programming-based method. State queries have been constructed on the basis of expert knowledge in a fuzzy logic system. A set of planning parameters have been required to be controlled over a planning horizon. At first, constraints relationships among query variables and planning parameters have been given then control parameters responsible for decision-making have been derived by reformulating fuzzy states as nonlinear constraints of a mathematical programming model. The adopted fuzzy methodology has enabled expert knowledge to be handled practically into a single method compatible with decision-making, comparing to traditional approaches, which have been quite often found individually or with no exact procedural guidelines.

The aim of this paper is to provide support for risk management decision-making by proposing a generic framework and model. Nowadays, more and more applications where risks related to variability and fuzzy uncertainties play an important role have been appeared in various curiosity areas such as telecommunications, software development, financial markets, medical imaging, supply chains, etc. Due to associated decision-making difficulty and complexity, the question of how best to support this kind of decision-making is a major concern for researchers. The aim is to provide a background and directions on this expanding area of research and application and to present an overview of some major developments based on this framework and model. A fast-growing trend of applying qualitative and/or fuzzy approaches in supporting risk management is expected. Some methodologies and tools that have been developed in this arena have been reviewed. The essential components to support risk management decision-making have been identified in. This paper is a repeat of it with a summer of recent works done mostly within the adopted framework and model.

7.2. Project Management

Project management has become a priority component of efficient management in various fields of activity. Project management flexibility is described as the ability of project managers to respond quickly to a threat or opportunity. It provides a competitive advantage of the project over its competitors in the same market. Project management flexibility is encouraged by the evaluation of project effectiveness, especially in the context of risk, uncertainty, and decision-making. A large number of developed methods for project effectiveness evaluation assumes satisfaction of analytical and statistical conditions that may not always be implemented (et al., 2019). The complexity of the project is not easy to define and relies on the unique circumstances of the scenario. However, several pragmatic dimensions can be used including, for example, number of person days, number of components, and staff turnover. It is assumed that the more complex a project is, the greater its chance of evolution into a complex state, and vice versa. Complex adaptive systems are pragmatically described as systems consisting of many components interacting with each other in a complex way. Such systems display structures, patterns, and properties that emerge as a result of the interaction of the components and evolve over time through competition, cooperation, and learning based on the interrelationships. Complexity, as a natural property of the world, has co-evolved with more traditionally studied phenomena such as perception, cognition, and sociality. New complex systems definition contributes to a growing body of research that delves into consequences and applications of this perspective on managerial, design, and organizational issues. Fuzzy logic in investment project management optimization is offers on the basis of the theory of fuzzy sets. Fuzzy set theory is fundamental for the theory of fuzzy logic. A fuzzy set with respect to the universe is a set containing a map that associates to every element of the universe a degree of membership in the [0, 1] interval. The membership function graph can take various shapes. Intuitively, fuzzy logic is an extension of Boolean Logic, which is a formal system focusing on truth values True and False, to the interval [0, 1] whereby reasoning based on degrees of truth is made possible.

7.3. Supply Chain Management

The competitive challenge faced by industries in the present globalized environment has compelled industries to give serious thought about their Supply Chain Performance. The model used had a huge number of input parameters which were defined by experts and a huge knowledge base was developed. Real time data acquisition and a review of the model created over a period of time to incorporate more input parameters. The world perceived the enormity of potential threats posed by the various networks on humanity and the environment. The prime objective of this research is to develop an inclusive approach for assessing the performance of closed-loop supply chain networks that would feature a platform for assessing: operational performance, efficiency and functionality of reverse logistics networks in industries with cash or new market competition; in competition with existing markets with rebate markets; in competition with existing markets with risk-reducers in the recycling industry. It would break new ground to farthest reach the sour point in developing nations where, if well managed, retrofitting industries would serve price, environmental, and social responsibility forces or, if neglected, would lead to serious threat to the ecosystem. There is a multitude of performance factors that vary with respect to industry type and nature of working that plays a vital role in closed-loop supply chain management. This triggers multiple searches for proper customization of the rule engine, inference engine, degree of fuzziness, levels of membership functions and output approximation on the forecasting front. The development of a hybrid between metaheuristic and fuzzy logic is worth exploring in extensive research especially in the domain of decision trees. Simulation of closed-loop cascade or supply chain networks with local governments and with tax producers would reveal the macro perspective for more responsible recycling assessment of performance in developing nations.

Product recovery such as refurbishment, remanufacturing, and recycling through reverse logistics networks has become a crucial aspect of supply chain management. While supplier selection and supplier performance assessment in closed-loop supply chains have attracted attention, only fuzzy logic based techniques have been employed to enhance performance through expert opinion aggregating systems. Multiple criteria based fuzzy decision support systems are required to enhance the feedback loop of monitoring process performance in collecting and storing operations. A hybrid between fuzzy and historical data based systems has the potential to develop into an expert decision support system to enhance remit engineering in damaged product assessments. For performance evaluation in collection of end-of-life products across polychronic and short horizons, network optimization based scheduling would lead to extensive research.

8. Case Studies

In this part of the article, we will study two of the conducted prototypes of the optimized fuzzy logic model. As for State1, Machine1 refers to the Agitation1 and Machine2 to the Heating1. For State2, Machine3 refers to the Agitation2 and Machine4 to the Heating2. State3 refers to the Loading and Unloading machines. Finally, Tank1 and Tank2 refer to Sensors 1 and 2 respectively, which can deliver a normal state or an alarming situation. The generated fuzzy logic model can output several states according to the internal state of each machine. If machines are in an empty state, the next possible state should be starting. If machines are still under construction, the output should be Busy. And if machines processed well and the cutting product has been achieved, the next output will refer to the clearing machines state.

The linguistic variables correspond to the state of machines. After defining intervals, we checked each interval for inclusions with other intervals. Intervals inclusion can help optimize fuzzy logic rules. Given that we aim to generate more than one output, our model can generate possible machine states in real-time and gives the decision-making step for the agents. This will minimize time, costs, and resources. Moreover, it is important not to eliminate the human factor; there will be collaboration between human and machine capabilities. Each row of the truth table represents a rule of fuzzy inference, containing one possible configuration of the input and output variables. The idea is to optimize the generation process by ensuring complete and fast fuzzy rules based on logical evaluation rather than linguistic rules. The fuzzy rules base consisted of 7 optimized rules based on the table. In the defuzzification phase, the system examines all of the rule outcomes after they have been logically added and computes the final output value of the fuzzy controller. Our experiment protocol aims to answer the following questions: Can the inclusions and intersections impact the outputs of the fuzzy logic controller? How can we generate optimized fuzzy logic rules intelligently? Which one of the fuzzy logic sets is the best for our case?

8.1. Case Study 1: Application in Healthcare

Healthcare systems and processes can be seen as soft rule-based control systems that select and apply the operations needed to meet the needs of patients. A knowledge-rich Decision Support System (DSS) within such a control system can select and implement a likely optimal sequence of the operations, given the current state of the healthcare system. However, the knowledge needed is difficult to acquire and formalize because of the uncertainty, vagueness, incompleteness and immeasurability of relevant information. Techniques for reasoning with uncertain and imprecise information have been developed over the past 20 years. This paper describes the use of such techniques to construct a DSS for a residential aged care service system. The expert knowledge needed by the DSS is mostly expressed in a fuzzy knowledge base. This includes rules for deciding class membership of residents who have not yet physically entered the facility and rules for determining personalised care plans with imprecise membership values. A prototype rule-based fuzzy reasoning system based on this knowledge base is currently being constructed (Warren et al., 2000).

Some practical issues must be resolved in order to assure the efficacy of the reasoning system. The first is deciding how to enter the knowledge into the knowledge base. Fuzzy sets can be a good measure of knowledge in this respect. The other issue concerns whether a fuzzy rule-based reasoning system can match a Complex Event Processing (CEP)-based knowledge base to enable real-time DSS operation. For operations pre-requisite and resource aggregation, a fuzzy event calculus based on fuzzy logic programming is constructed, providing fuzzy states which represent how operators, if executed with degrees of fuzziness, would affect the state of enabled operations (Kerarmi et al., 2022).

Based on these measures, an online reasoning engine for real-time front seat agent detection and classification can be developed, which runs with car-mounted cameras. Finally, a novel temporal fuzzy restriction in the fuzzy event calculus is employed to represent the fuzziness of real-time conditions occurring to a mobile environment. An example of intelligent real-time detection and classification of occupied vehicles and car make models are presented.

8.2. Case Study 2: Application in Finance

The financial systems in which all types of trades are tracked are implemented in the managing systems of each financial institution. These management systems have three main modules based on uncertainty states: modeling of fussy uncertainty state manager, fussy uncertainty state manager, and stochastic uncertainty state manager (O. Weber et al., 2007). A case study for each uncertainty state management module is presented. The sub-case studies, in turn, are based on real data, which are fully accessible. The functional requirements and design of the modules are extensively described, followed by suggestions for their further development. In assessing any asset inefficiencies of financial sources, experts or computerized assessment approaches define the characteristics of future possibilities. The asset inefficiencies can be assessed by some previous cases occurring in time, risk variables, and management rules. The variables that affect risk states of assets are widely modeled with some approximate functions, such as linear, polynomial, exponential, logarithmic, or sigmoid ones.

Most widely used fuzzy systems include Mamdani (using fuzzy if-then rules) and Takagi-Sugeno (TS) models (using fuzzy if-then rules and a defuzzification procedure). Although Mamdani fuzzy systems are more intuitive and support a richer set of rules, TS fuzzy systems being of the higher generalizes and potentially more powerful postponed function identification) are considered to be more appropriate than Mamdani fuzzy systems for financial applications. Several works have studied the use of fuzzy systems for risk analysis modeling in finance (Serguieva, 2004). For instance, fuzzy systems are applied to estimate the risk of individual derivative contracts such as interest rate swaps, swaptions, and caps. A case-based reasoning framework is proposed to model uncertainty of default risk in corporate bonds. In another work, fuzzy systems are adapted to evaluate conditions under which an exchange rate attempts to complete sudden short-run downtrend movements. On the other hand, a large number of financial strategies under uncertainty are modelled by stochastic optimization. While fuzzy systems handle uncertainty with words, stochastic models deal with uncertainty with random variables and probability distribution functions.

8.3. Case Study 3: Application in Environmental Science

In recent years, great efforts have been made to improve understanding of environmental performance and the factors that contribute. This has prompted socially responsible action by many organizations, as environmental performance is becoming an increasingly important issue in today's society. Legislation has led to the growth of environmental management systems. Organizations are therefore keen to evaluate their environmental performance, where the results from this assessment may be used to advise improvements to existing practices (Humphreys et al., 2006). As there is often a large amount of uncertainty in measurements and indices when assessing environmental management systems, this evaluation process may involve the application of fuzzy logic. Performance assessment shall therefore encompass a wide range of subjective as well as objective evaluations of environmental performance. Metrics in terms of fuzzy variables with some degree of uncertainty shall be utilized.

Assessing the risk to pollinator populations from complex stressor interactions is a major challenge faced by regulators and industry, primarily due to the number of parameters of concern. Threshold values have been established for certain toxicants and behaviors, but the interactions between the variables are complex, so these thresholds are not universally applicable (Bassford & Painter, 2016). Fewer studies have examined the interacting effects of predators, parasites, pathogens, immunosuppressants, and changes in foraging or nesting behavior either on their own or together within a constructed bio-environment in vitro. The aim of this project is to develop a method for accumulating monitoring data for bio-environments, identifying potential future stresses from previous inputs, and appropriate locations for their implementation to maximize the chance of observation.

9. Challenges and Limitations of Fuzzy Logic

Fuzzy Set Theory has been developed during the last decades to a demanding mathematical theory. First introduced in 1965 by Zadeh, it is based on the premise that all terms in a natural language are fuzzy rather than crisp. It involves degrees of set memberships rather than precise memberships. A set A is fuzzy if it is associated with a membership function defined in the range [0, 1]. Real applications are normally not single-method applications but are rather complex combinations of different techniques. Some methods combine together to gain more accuracy than when applying them individually. Basically it is useful when dealing with imprecision and it does not require statistical data to model the system in an understandable manner. For example fuzzy linguistic approaches for modeling supply chains are feasible when suppliers, manufacturers, trainers and even goals are not sponsored by a set of crisp numbers but rather by fuzzy verbal labels ('very few suppliers', 'not critical', 'price is very cheap' etc.) (Meyer & Zimmermann, 2011). The fuzzy set A and its membership function usually denoted by ?, are used to formulate the concept "A is fuzzy". The modified fuzzification of the concept introduces a class of fuzzy (??) sets A which are better than"i for interpreting it.

The purpose of this paper is to discuss the applicability of fuzzy logic over crisp logic in generating intelligent response for natural language inputs. The opinion on the conclusion of effectiveness, efficiency and robustness of fuzzy and crisp logic in their own modes of application is also dealt with (V. Prasad et al., 2012). Human beings handle most situations with such incompleteness and imprecision robustly. Perfection cannot be attained in such cases. A humanoid robot mimicking a human being is a culmination of life sciences, neurology, pattern recognition, linguistics, engineering, robotics, graphics, mathematics and many more fields which is yet to be assembled. If a robot being endowed with all these fields of knowledge in addition to perfect data about the macro and micro world including human beings, its own structure and mechanism itself.

9.1. Computational Complexity

Defuzzification is the process of translating the result of fuzzy inference, a fuzzy set, into a crisp number. In general, defuzzification involves the same exact reasoning steps as fuzzification, but instead of replacing the crisp values by vague linguistic values, the final conclusion or result is expressed in terms of a particular crisp output variable. All the information remains in the fuzzy domains of the linguistic value and the variable, which have to be translated back into conventional numbers. Though defuzzification is perceived as an overall translating process, it should be noted that it doesn't call for new mathematical algorithms related to fuzzy logic itself, rather, it involves existing mathematical techniques of non-fuzzy set theory. Defuzzification algorithms can be designed in several ways: as single-stage defuzzification algorithms (in which numerical input variables are directly transformed into numerical output variables) and as successive defuzzification algorithms (in which, intermediate fuzzy set manipulation is permitted) (F. Eick, 2013).

Principally, defuzzification can be steps like composing fuzzy sets, union operations, cut operations, threshold operations as discussed earlier in the fuzzification process, numerical operators as addition, multiplication, division, negation, >(greater than), (less than), (greater than or equal to), (less than or equal to) transforming syntax values or parameters to controversy values and the other way around. This section focuses on reviewing single-stage algorithms, including the center of gravity (also known as centroid; center of area/polygon) method, maximum membership method, center of maximum method, leftmost method, and rightmost method. Probabilistic fuzzy numbers are fuzzy numbers with the additional notion of probabilistic membership. The membership functions of probabilistic fuzzy sets are represented as random variables; in general, they are expressed in the forms of dual gaussian (or normal) distributions. (Ridvan Benatar, 1970) developed different techniques to determine the corresponding membership functions of probabilistic fuzzy sets.

9.2. Interpretability Issues

Some major criticisms of fuzzy systems are that they can work as black boxes and that they do not take into account the current state of uncertainty in their reasoning. Therefore, the overall goal of this chapter is to show how crisp rules may be generalized to fuzzy rules in a way that is computationally efficient, meaningful, and interpretable, while also addressing how a process can manage and utilize the current levels of uncertainty on the available input information for reasoning purposes in fuzzy expert systems.

First, it is shown how to generate interpretable fuzzy if-then rules from a knowledge base of crisp if-then rules as given by domain experts. The interpretable fuzzy rules are in the form of fuzzy-sets composed of piecewise linear fuzzy sets with two different slopes. This way of interpreting the existing implicit knowledge allows the fuzzy systems to be more easily understandable than current fuzzy rule-based systems, as they are based on simple triangular shapes. The method is demonstrated on a crisp knowledge base composed of rules in the form of intervals. Also, it is shown that the meaning of the rules is preserved after the generation process (Bodenhofer & Bauer, 2005). The overall reasoning can be executed by means of Mamdani systems.

Second, reasoning with degrees of uncertainty on both the fuzzy sets and fuzzy rules is addressed, by generalizing the original notion of fuzzy reasoning from the classical degrees of truth and interpretation to focus on their role in handling an uncertainty state. To do this, qualitative fuzzy variables are taken into account, as well as a current state of uncertainty modeled as a randomness probability distribution on the degrees of fuzziness of the fuzzy sets in the knowledge base, and on the degrees to which the fuzzy rules are considered relevant for reasoning in the current state of knowledge (H. Ruspini, 2013). This way, fuzzy sets, numbers, and variables are generalized from crisp values to a distribution of values in a more general framework of fuzzy and probabilistic reasoning systems. The mathematical foundations for this extended concept of fuzzy reasoning are provided and some examples are shown to illustrate how it could be utilized in applications. In this chapter, the approach is mainly illustrated in the context of Mamdani fuzzy systems.

10. Future Directions in Fuzzy Logic Research

Fuzzy set theory and fuzzy logic have proven effective for the management of uncertainty states. There already exists a vast number of applications and software implementation with demonstrated utility across various social fields, including engineering, business, and other realms in a multitude of forms. Nevertheless, the theoretical development of fuzzy set theory already demonstrated many facets yet remains rather immature. Even though the state-of-the-art fuzzy technologies provide a typical set of adequate tools for the modelling of uncertainty states within engineering or managerial settings, there is still much room for improvement. It is hoped that researchers in the field of fuzzy logic pursue alluring questions of theoretical or methodological longevity and that, at same time, many ingenious applications be devised within engineering and development organizations that lead to important social or economic benefits (Meyer & Zimmermann, 2011).

Many conflicting and incompatible definitions of the notion of fuzzy number have been found in the fuzzy literature. The goal of the research was to compare a few of them and to illustrate that one of the definitions has some shortcomings in the crisp case, while some generalizations proposed by the conflicts retain all the usually demanded properties of the meaning of fuzzy numbers. It is concluded that the lack of an international standard allows this heated debate to continue and likely even grow. Moreover, the necessity for an international body to address fuzzy number and fuzzy set issues is underlined by the increasing emergence of alternative fuzzy paradigms and, worryingly, the absence of a clarification of existence, necessity, and meaning of fuzzy set and fuzzy logic theories and ideas in view of the ever-broader usages and implications of fuzzy set-based technologies (J. Liberatore, 2016).

10.1. Integration with Machine Learning

The relation between machine learning and fuzzy systems has been studied extensively in recent years. The fuzzification of data may help machine learning or stochastic models to better deal with uncertainty. For example, one of the ideas behind robust fuzzy classification is that a soft classification, which is based on continuous values, may outperform hard decisions. Also, it is likely that data augmentation can help classification algorithms when the original dataset is not large enough and/or representative enough. Several data augmentation methods based on fuzzy systems were proposed. On the other hand, fuzzification of models turns the determination of their parameters a highly non-linear optimization problem, but evolutionary algorithms and similar machine learning methods can be used to search for a good solution. Finally, the use of fuzzy models to interpret and explain the predictions of certain machine learning methods is also an interesting and valuable research topic that has attracted the attention of a few researchers. Given the large overlap between fuzzy systems and soft computing, the fuzzification of wavelet networks, recurrent neural networks, and radial basis function networks has been studied in the context of function approximators. The relationship between fuzzy systems and rough set theory has also been investigated. Additionally, fuzzy hybrid intelligent systems combining fuzzy logic with other paradigms such as genetic algorithms, neural networks, ant colonies, particle swarm optimization, support vector machines, or ELM have shown great success. Apart from this specific integration, hybridization in general, where different paradigms are combined, has witnessed increased interest and research activity. The objective is that hybridization may yield a better fit system than merely using independent systems. Either a loose framework of independent components is used or the outputs from different components are combined in a more or less sophisticated way (S. Smith, 1990). Other possibilities are feed-forward architectures, where the output of fuzzy systems feeds back into other systems as input variables, or even feedback mechanisms where the components interact continuously over time (Kerarmi et al., 2022).

10.2. Advancements in Fuzzy Algorithms

Advancements in fuzzy algorithms include generating fuzzy rules by learning from examples, enhancements in Karnik–Mendel algorithms, and developments in interval type-2 fuzzy logic systems. Fuzzy rules can be generated automatically from numerical input/output data using linguistic rule and boundary assignments with a combination of fuzzy techniques. Enhancements in Karnik–Mendel algorithms include substantial reduction in the number of polygon area calculations and the use of reduced fuzzy sets in the adjustment process. Furthermore, walker, via reparameterization transform innovations, and extended initialization methods, has expressed the type-reduction problem in fewer than 16 edge image manipulation functions. An interval type-2 fuzzy logic system cannot be implemented by traditional type-1 fuzzy logic systems. Therefore, alternative type-reduction approaches have been adopted to reduce the computational costs of interval type-2 fuzzy logic controllers, able to process in real-time. Recently, uncertainty bounds have been used for designing the interval type-2 fuzzy logic systems based on the boundary locus. Fuzzy sets, which are a generalization of classical sets, have led to the born of new mathematical objects as fuzzy relations, fuzzy numbers, fuzzy logic, etc. After introducing the concepts of fuzzy propositional logic and fuzzy predicate logic, the connection between fuzzy logic and fuzzy sets has been identified through logical calculus defined on fuzzy propositions. All those concepts have been successfully applied over the last years to modelling and control and also to the specification of fuzzy knowledge. These concepts are basic and mature, but there is still a trend of construction of new ones, such as fuzzy integrals, measure of fuzziness, fuzzy measures. On the other hand, linguistic variables together with their qualitative values can be colloquially regarded as words and meanings ascribed to them. Fuzzy sets formed on linguistically expressed information can be considered as a bridge allowing to transfer qualitative evaluations into the quantitative language of control. The seminal and pioneering work of zadeh has already opened a new area of investigation on the interaction between fuzzy sets and linguistic variables. Nevertheless, there still remain many challenging and these problems are expected to inspire fresh innovations and creativity in the 21st century (Ridvan Benatar, 1970).

11. Conclusion

Fuzzy logic seems relevant and appropriate to cover the qualitative viewpoint regarding PMC in uncertain states. Fuzzy logic provides a framework, which can manage uncertainty in continuous values instead of exact crisp boundaries. Uncertainty or imprecision in states of models or machines causes difficulties in system management. This can be due to the complexity and non-linearity of working conditions, or the human effect in measuring states. Managing systems with uncertainties state is a challenging task as vagueness in measurements can lead to vagueness in outputs too (Kerarmi et al., 2022). A fuzzy set consists of a collection of elements that have varying degrees of membership in the set. The operational fuzzy logic can manage fuzziness in real-world applications. Defining fuzzy rules can help to manage the states of an evaluated system. To manage a system with different states, specifying fuzzy rules is an agreeable way. The fuzzy rules can be generated in two manners: 1) Expert-fuzzy rules which are built from experts 2) Evaluated- fuzzy rules which are generated from regressive algorithms. The analysed fuzzy rules to manage the generated uncertainty states of the system clarify how the proposed model would perform. The analysed rule clearly shows the effect of managing rules to control uncertainty in decision-making. Qualitative variables of models or machines are naturally fuzzy, so fuzzy logic techniques are appropriate to manage the qualitative and uncertain aspect of the variables (Gurrea Montesinos et al., 2014). The approach of using fuzzy rules gives a qualitative view of modified quantized variables in the system that is effective in controlling PM states. The advantages of the approach are that the fuzzy rules generate in overlapped method that makes decision more flexible. This gives a chance with qualified accuracy for misverifications. Its flexibility can help in better work with human abilities too. Because of overlapped conditions some fuzzy rules are out of conditions but this almost always will not cause selection failure. Handling uncertainty in the state of discrete variables using qualitative fuzzy rules is usually difficult through direct application of fuzzy rules.

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